FORUM

MATHEMATICS ON PROBATION

By Serge Kruk

"I heard that the Mathematics department of Oakland University is on probation."

The first time I heard this nonsense, related to me by a colleague over lunch, quoting a student, I internally laughed at the fools who would believe such drivel. I pictured each of my colleagues in turn, standing at the corner of a nondescript classroom, facing the wall in penance, with a pointy hat on their head. The image still amuses me, but I have heard this canard repeated so many times that I lost count, as well as my capacity to laugh at it. Over the years, I heard it mostly from students who trusted me and were inquiring honestly, if naively, about its truth. They had heard the statement, repeatedly, from other students and seemingly could not dismiss it out of hand. Since they were honest inquiries, I responded in kind. Seizing an opportunity to instill some critical thinking, I briefly described the work of a mathematician in a research department. I spoke of research, teaching, service and the delicate balance between competing goals. I described the peer-review process and then I asked the student to imagine which body, using which criteria, and by what authority, would "put a de-

partment on probation.“ Most of my students eventually got the point and smiled at their own gullibility.

Recently, the source of the offending statement changed. I read it in a letter from the angry mother of an Oakland student, berating all mathematicians because her cherished progeny had failed a mathematics course. Her child had taken some mathematics at colleges before and had never, ever, failed a course. “No wonder we were on probation,” said she. Over the years, my original amusement at the fools who put faith in such rumours turned to anger at their stupidity and, finally, to puzzlement at the virulence of the meme. It would be a fascinating psychological essay to study its origin and spread. But my interest today lies in a discussion of the possible rationalizations people might construe to support the belief that Oakland’s mathematicians are so incompetent as to be “on probation,” whatever that could possibly mean. I will offer an alternative explanation to the rationalizing logic and consider some of the negative effects of this nonsensical belief.

At a bird’s eye view, mathematicians at a Ph.D. granting institution spend their time in three tasks, in decrease order of importance: research, teaching and service. In a nutshell, publishing papers to advance the state of knowledge, educating the young in the basics of our discipline, and wasting interminable hours in committee work to appease the gods of bureaucracy. In the context of rationalizing “probation,” we can certainly ignore the third component of the work, the so-called service, on the observable fact that no level of incompetence in service would ever be held against a faculty member. In fact, a good case could be made that incompetence in service is rewarded: we never call on the incompetent (except, sometimes, to promote them to administrative position; but I digress). As for research, it is largely ignored or unknown by the general population. Students and their over-protective mothers can be forgiven for ignoring mathematical research: it has no effect on their lives, not unless they become graduate students. Maybe we could fault certain mathematicians for pursuing obtuse and useless research agenda, but I suspect that irrelevancy

of research is not cause for "probation." We are thus left with teaching as the only aspect of our work whose scrutiny might warrant the probation myth in the gullible mind.

How can we rationalize the probation myth? Assuming that its foundation relies on some facts, not fancy, we have one indubitable fact that could be harnessed to the cause: students fail mathematics classes. At Oakland, in some Freshmen classes, the failure rate can reach 60%. (Note that this does not apply to upper-year classes; most of those enjoy almost perfect success rate.) The abysmal outcome in lower-level classes is consistent, semester after semester, instructor after instructor. Does it lead to the conclusion that all thirty or so mathematics instructors at Oakland are all incompetent (not to mention the additional scores of mathematicians that journeyed through Oakland on their way to other institutions)? The student's mother, cited above, was clearly linking her daughter's failure in mathematics to our failure as teachers. Is it sound to infer incompetence from failure rates? Let me ponder this and try to enlighten the discussion.

In lieu of introduction, let me mention one fact that is rarely mentioned in the failure rate discussions: the unusually comprehensive chain of pre-requisites courses that is attached to each mathematics class, at Oakland and elsewhere. I will not claim to know the whole Oakland catalog and there may be other departments, out there, with long chains of pre-requisites courses, but typically the chain is short. One course or two opens the door to most or all junior-level courses. Contrast this with the pre-requisites in Mathematics: MTH011 to get to MTH012 to get to MTH121 (or STA225 or MTH141 or APM263) to get to MTH122 (or MTH154) to get to MTH155 to get to MTH254 (or MTH255 or MTH275) etc... All this before the first 300-level course and all of these compulsory for some non-mathematical discipline (Business, Engineering, Computer Science, Nursing to name a few of the numerically important ones). This chain goes on and on for all courses, at all levels. Moreover, these pre-requisites are real. They are not stated simply to herd students into taking courses for the sake

of optimizing course offerings, but rather because we rely, in an essential way, on the vocabulary, concepts and techniques of one course to pursue the material of the next. (The proper analogy, if one is needed, is to language courses: One does not start Russian 101 by reading Pushkin). This is not set in stone, of course. We could have decided that some material should be presented in a different order. Mathematical knowledge is so profoundly interwoven that we could unravel the cloth starting from different strands. But, once we choose an approach, the chain is compulsory and enforced. If a student cannot factor polynomials (a pair of MTH012 concept, "factor" and "polynomials," with a small but non-trivial set of related techniques) he will likely fail in MTH141. There is no way out! Note that if he did not learn these previously introduced concepts and techniques, he might, conceivably, learn them in MTH141 (If the political poetry of Pushkin is attractive enough to a student exquisitely sensitive to the slavic soul or to oppression, no grammatical barrier is unsurmountable, I suppose), but this means an additional burden on the student's limited time and commitment. Hence if a student fails to master the material of a course, he will struggle in the next, simply because he will not understand a good deal of the material, lacking the vocabulary, and will mishandle the new techniques, because they require mastery of, not simply acquaintance with, the old techniques.

Let us now consider whom we teach. Oakland graduates about a dozen mathematics majors every year, yet we teach thousands of students per year. Mathematics professors at Oakland by and large do not teach mathematics majors. We teach engineering, business, nursing, education and science majors. In the consecrated expression, we teach "service courses," id est, courses for students of other departments, courses that serve as foundations for either subsequent work in specialized fields or courses meant to build a well-rounded general education. In this regard Oakland is not unusual. There are very few places like my Alma Mater, with five thousand undergraduate students in mathematics and three hundred mathematicians

teaching exclusively mathematics majors. So at Oakland, as at most universities in this country, every semester, mathematicians teach a few thousand students everything from remedial high-school material to the elements of real mathematics. Most students, from the start, resent being in remedial classes or, equivalently for our purposes, resent having to do mathematics before they get to their chosen subject ("I'm going into Nursing—or Elementary education, Engineering, Business. Why do I need this stuff?"), an attitude unlikely to foster interest and good work habits. So we teach students who do not want to take our classes.

It gets worse. Let us consider what we teach in more detail. I read with pleasure tinged with envy *The Oakland Journal* article of the Fall 2007 issue. Karen Miller, commenting the first years of Oakland University, describes an enlightened requirement: "All students were required to demonstrate proficiency in calculus and complete two years of training in a foreign language." The founders of Oakland had in mind a true Liberal Arts education, a broad study including foreign languages. How I would have loved to converse with my students in something other than American English! Calculus, in this context is nothing but another foreign language, that of physics. Let me stress that this subject is not a new and obtuse development of modern mathematics; it is an eighteen century language developed to understand accurately and describe concisely the movement of bodies. Understanding calculus means understanding that the speedometer in a car is computing a derivative; it means understanding exactly what is meant by "The rate of change of temperature." It means understanding the movement of the earth and its effect on the seasons. These are basic, essential concepts in our technological world. To understand more modern contraptions, say computers and the internet, we should today add to the required languages those of discrete mathematics: enumeration, graph theory, probability. Alas, instead of adding, we subtract. We have abandoned all but the pretense of a comprehensive liberal education. To wit: In 1958, calculus for all! A few years

later, less than stellar success rates forced Oakland's mathematics department to introduce pre-calculus, but with the clear aim to get students to understand calculus. Today, we have pre-calculus and pre-pre-calculus and pre-pre-pre-calculus, three levels of remedial mathematics. Moreover, these remedial classes, covering middle and high school material, are not stepping stones to calculus anymore; they are the end of the mathematical journey for many students.

How many students are at remedial mathematics level? Here is a back-of-the-envelope estimate: Oakland admits about four thousand new students per year. We teach over two thousand students in remedial classes every year. Inescapable conclusion, especially since many students need no mathematics in their program: the majority of Oakland students start their university career with a deficit in mathematics. Judging by the standards of the founders (calculus for all), we wade in a morass of incompetence. At Oakland, we do not give credit for the first two levels of remedial classes but we do for the third, mostly for base political reasons, not because any of us deem the material worthy of university credit. How can students possibly react, except with resentment, to be herded in a remedial class, covering material that they have already covered in middle and high school, material that they think they know? The obvious falsity of the latter belief is manifest by their failure in the placement test, yet reason cannot prevail and we try to teach them in one or two semesters, what they failed to learn in the previous three or four years of education.

Why do we teach these remedial classes? Aside from having abandoned the ideal of our founders, it is because at Oakland, as in most other universities in this country, we have recognized the fact that students graduate from high school without the proper mastery of the mathematical material prerequisite for university-level work. We judge this inadequacy with a number of tools, among them a placement test developed by the Mathematical Association of America, a placement test failed by most of the Oakland students who attempted it in 2006. (We are not alone in this situation; remedial classes are

offered almost everywhere. Of course, at the University of Michigan, they can get away with one level of pre-calculus to our three. I leave it as an exercise for the reader to conjecture why. Here is a hint: "Oakland University must have 20,000 students by 2010".) Remedial mathematics classes in all universities in the country are a clear-cut case of the dumbing down of America, if ever there was one. While, in 1958, humanities students needed to pass a calculus course to prove their membership in the ranks of the intellectually enlightened, today many programs contend with middle or high school level mathematical knowledge; watered-down curricula that makes it nearly impossible to understand the assumptions buried in a graph of rising temperature, or the effect of anti-depressor drugs versus a placebo, or the effect of teaching methods on success rates, for that matter.

In the interest of honest debate, let me admit that I am neither surprised nor ashamed by a 60% failure rate in remedial classes. How can we teach in one or two semesters what students failed to learn in years of schooling? This admission will immediately classify me, by some, as a heartless snob who cares not one wit about his students. Yet I do not want an elementary education or a nursing student to be so ignorant of basic mathematics that she (likely she, not he, even in our post-feminist era) cannot understand the demagogical claims surrounding global warming or big-pharma's claims about the efficacy of anti-depressors. In the twenty-first century, a scientifically educated polity is a necessity. Furthermore, if tangentially, let me point out an important and known, if not well recognized, fact: students do not fail the whole of their university studies when they fail a math course. But if they fail at a large number of their course, they invariably fail at math. By our position as gatekeepers for many programs, we are first to diagnose intellectual weaknesses. (In contrast, students on academic probation, the probation state that really exists and handles hundreds of students, each year, even if they fail almost all courses, mostly manage to pass Rhetoric Freshman classes. Considering the writing skills of some of my students in

the only language that they usually claim to know, I wonder if my colleagues in Rhetoric abandoned the fight.)

There are innumerable reasons why students fail in mathematics; I have not covered them all. But the context of the failure has to be informed by whom we teach, what we teach, by the long chain of pre-requisite vocabulary and techniques and, also, by the particular flavour of anti-intellectualism that pervades our society. Let me summarize: Mathematics courses rely, in an essential way, on the vocabulary, concepts and technical fluency of the courses sequentially preceding. Each semester, we teach thousands of students who would rather not take any mathematics at all. We teach them remedial material they failed to master in middle and high school, material they resent having to study again, to study at all. The students' negative attitude is reinforced by a pervading anti-scientific cultural atmosphere. The stage is set for a disaster. The disaster occurs: semester after semester, an army of students fails introductory mathematics courses. Are we surprised? I am not. Neither am I on probation.